# Optimum Filling of Ferrite Phase Shifters of Uniform Dielectric Constant

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*Abstract*—Approximation methods are used to calculate the phase shift and loss for phase shifters containing ferrite and dielectric, with a uniform dielectric constant throughout the waveguide. If the RF magnetic loss in the ferrite is a significant fraction of the total loss, the overall performance of a phase shifter that is partially filled with ferrite may be superior to the fully filled case. Theoretical results relating performance to the amount of partial filling are presented for Faraday rotators in square and circular waveguides and a twin-slab phase shifter. Experimental results were obtained for a circular Faraday rotator.

## I. INTRODUCTION

A FERRITE phase shifter that employs a waveguide completely filled or almost completely filled with ferrite offers a small cross section that is compatible with lightweight phased-array antennas. Analysis of structures in which the dielectric constant is the same both inside and outside the ferrite region indicates that a partially filled structure may provide performance that is superior to that of a fully filled structure. In the latter case, certain regions of the ferrite may contribute little to the phase shift but will make a significant contribution to the overall loss as a result of RF magnetic loss in the ferrite. By replacing the ferrite with dielectric in these regions, the loss may be reduced considerably with little degradation in phase shift.

Approximation methods are used to calculate the differential phase shift and magnetic loss for the dominant transverse electric (TE) normal modes in the waveguide. Combining the magnetic loss with the dielectric and conductive losses, a figure of merit is defined as the ratio of differential phase shift to total loss, normalized to the case where the waveguide is fully filled with ferrite. Applying this method to several useful configurations, the figure of merit and optimum partial filling are presented. The specific structures considered here are Faraday rotators in square and circular waveguides and a twin-slab phase shifter in rectangular waveguide (Fig. 1).

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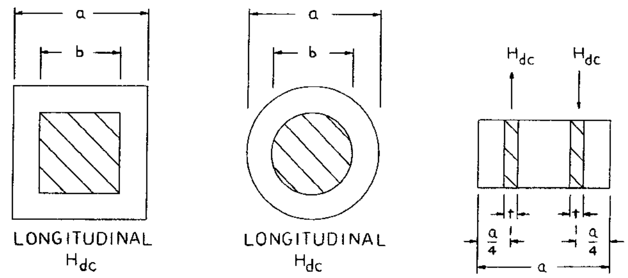


(a) (b) (c)

Fig. 1. Cross sections of waveguide phase shifters. (a) Square Faraday rotator. (b) Circular Faraday rotator. (c) Twin-slab phase shifter. Ferrite is indicated by cross hatching; the remainder of each waveguide is filled with dielectric.

## II. BASIC ASSUMPTIONS

In order to calculate the phase shift and losses, the following assumptions are made: the dielectric constant $\epsilon = \epsilon' - j\epsilon''$ is the same in the ferrite and the dielectric regions; the ferrite is weakly magnetized and operation is at biasing field levels far below magnetic resonance. For a dc magnetic field in the $z$ direction, the RF permeability of the ferrite is characterized by a tensor:

$$\bar{\boldsymbol{\mu}} = \begin{pmatrix} \mu & j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & \mu_z \end{pmatrix} \tag{1}$$

where

$$\mu = \mu' - j\mu''$$
$$\mu_z = \mu_z - j\mu_z''$$
$$\kappa = \kappa' - j\kappa''.$$

For a low-loss ferrite of the type used in microwave components, the imaginary parts of the tensor components will be much less than the real parts and are ignored for phase shift calculations. However, since the domains in the ferrite will still be fairly randomly oriented when weakly magnetized and the ferrite magnetization will vary (including the state of no magnetization), an isotropic permeability $\mu = \mu' - j\mu''$ is used for loss calculations. This is reasonable in light of measurements by LeCraw and Spencer [1] of all six components of the permeability tensor for a typical magnesium manganese ferrite. At biasing field levels where the ferrite is unsaturated, they found that $\kappa'' < 0.05\mu_z''$ and $\mu'' > 0.85\mu_z''$ [1, figs. 6, 7, and 12].

## III. THEORY

### *A. General Considerations*

The differential phase shift per unit length $\phi$ is defined as

$$\phi = \beta_+ - \beta_-$$

where $\beta_+$ and $\beta_-$ are the phase constants for the normal modes of the particular phase shifter under study. The loss per unit length $\alpha$ is the real part of the propagation constant. The normalized figure of merit is then

$$F = \frac{\phi/\alpha}{\phi_0/\alpha_0}$$

where $\phi_0$ and $\alpha_0$ refer to the fully filled case. Note that $\alpha$ refers to total loss, resulting from magnetic, dielectric, and conductive losses.

To illustrate qualitatively why partial filling may improve the figure of merit, consider the Faraday rotator in square waveguide, Fig. 1(a). The normal modes for this structure are right- and left-circularly polarized waves or, equivalently, waves of a given circular polarization propagating in the forward and reverse $z$ directions. The mechanism for phase shift in the Faraday rotator is the coupling of transverse magnetic RF fields by the permeability tensor resulting from an applied longitudinal dc magnetic field ($H_{dc}$). The transverse coupling will cause the $H_x$ component to give rise to an $H_y$ component, and vice versa. The longitudinal component $H_z$ does not contribute to phase shift. However, the magnetic loss in the ferrite will result from the total RF magnetic field, including $H_z$.

Now consider the waveguide fully filled with ferrite. Since the circularly polarized modes are linear combinations of the $TE_{10}$ mode ($H_y = 0$) and the $TE_{01}$ mode ($H_x = 0$), it is evident that along the perimeter of the waveguide at least one of the transverse components of $H$ is 0. Under these conditions coupling cannot occur between $H_x$ and $H_y$. In the general region close to the perimeter the contribution to the phase shift should therefore be small. The contribution to the magnetic loss may be large in this region, particularly as cutoff is approached and $H_z$ becomes very large relative to the transverse $H$. Maximum contribution to the phase shift occurs, of course, in the center of the waveguide where the RF fields are completely transverse ($H_z = 0$).

If the ferrite is reduced in size by a small amount, by the reasoning above, $\phi$ will not change very much, but the magnetic loss may be reduced significantly. An increase in the figure of merit would then be expected, assuming dielectric and conductive losses to be held constant. Obviously, since $\phi$ must go to 0 as the ferrite is removed, the figure of merit should reach a maximum for some amount of partial filling. The same reasoning applies to the Faraday rotator in circular waveguide, Fig. 1(b), where the transverse magnetic field is comprised of radial ($H_r$) and azimuthal ($H_\theta$) components.

Similar arguments may be used to show that optimum partial filling exists for the twin-slab phase shifter, Fig. 1(c). In this case the dc magnetic fields are parallel to the $y$ axis. The resulting permeability tensor will couple the transverse ($H_x$) and longitudinal RF magnetic fields. The normal modes for this configuration are the $TE_{10}$ waves propagating in the forward and reverse $z$ directions. Maximum field coupling occurs in the vicinity of $x = a/4$ and $x = 3a/4$, where the RF magnetic fields are circularly polarized. No coupling will occur at the side walls of the waveguide ($H_x = 0$) or at $x = a/2$ ($H_z = 0$).

### *B. Loss Calculation*

For small isotropic losses, $\alpha$ may be expressed as the sum of the conductive and dielectric losses ($\alpha_{cd}$) and the magnetic loss ($\alpha_m$):

$$\alpha = \alpha_{cd} + \alpha_m.$$

For the fully filled case, let $\alpha_m = \alpha_{m0}$; then

$$\frac{\alpha}{\alpha_0} = \frac{\alpha_{cd} + \alpha_m}{\alpha_{cd} + \alpha_{m0}} = \frac{1 + rL}{1 + r}$$

where $r$ is the loss ratio: $r = \alpha_{m0}/\alpha_{cd}$; and $L$ is the magnetic loss factor: $L = \alpha_m/\alpha_{m0}$.

The power flow in a waveguide $P$ varies as $\exp(-2\alpha z)$ for a wave propagating in the forward direction. The magnetic loss is then

$$\alpha_m = \frac{P_{Lm}}{2P}$$

where $P_{Lm}$ is the power lost per unit length resulting from magnetic loss [2, p. 10]:

$$P_{Lm} = \frac{\omega\mu''}{2}\int_S |H|^2\, dS.$$

$S$ is the cross-sectional area of the ferrite. (Note: all calculations involving the RF magnetic field intensity use the form of $H$ from which the exponential dependence on $z$ and $t$ have been removed.) When fully filled, $S = S_0$ and

$$L = \int_S |H|^2\, dS \Big/ \int_{S_0} |H|^2\, dS. \qquad (2)$$

### *C. Phase Shift*

A perturbation method particularly suitable for this problem [3] is used to calculate the phase shift. This method is based on the use of operator methods in electromagnetics [4] by which Maxwell's equations are embodied in a single eigenvalue equation for propagation in waveguides.

The approximate phase shift is given by a first-order perturbation of the dominant normal modes. If $H^+$ and $H^-$ refer to the magnetic field vectors for forward (+) and reverse (−) propagating normal modes, the phase shift between the two directions of propagation is given by

$$\phi = \omega\mu_0 \int_S jH^- \cdot [(\bar{\chi}_m - \bar{\chi}_m{}^T) \cdot jH^+]\, dS. \qquad (3)$$

$\bar{\chi}_m$ is the magnetic susceptibility tensor from (1), and $\bar{\chi}_m^T$ is its transpose.

### D. Maximizing the Figure of Merit

The figure of merit may be put in the form

$$F(x) = \frac{1+r}{\phi_0} \frac{\phi(x)}{1 + rL(x)} \tag{4}$$

where $x$ is a geometry dependent variable (proportional to $b/a$ for the Faraday rotators or $t/a$ for the twin-slab phase shifter) representing the partial filling for the particular structure under consideration. To find the optimum filling, the figure of merit may be differentiated with respect to $x$:

$$F' = \frac{1+r}{\phi_0} \frac{(1+rL)\phi' - rL'\phi}{(1+rL)^2}.$$

Then for $F$ to be maximum,

$$(1 + rL)\phi' - rL'\phi = 0. \tag{5}$$

In general, (5) is a transcendental equation in the variable $x$ and must be solved using numerical techniques. The value of $b/a$ or $t/a$ corresponding to the value of $x$ that satisfies (5) is referred to as the optimum filling ratio for the specified $r$.

In order to illustrate graphically the one-to-one correspondence between $x$ and the parameter $r$, equally valid results may be obtained more directly by reversing the roles of variable and parameter in (5) and solving for $r$:

$$r = \frac{\phi'}{L'\phi - L\phi'}. \tag{6}$$

By choosing a value of $x$, the value of $r$ and the corresponding maximum figure of merit may be calculated in a straightforward manner from (6) and (4).

## IV. APPLICATION TO SPECIFIC CONFIGURATIONS

For the two Faraday rotators the tensor in (3) for phase shift is

$$\bar{\chi}_m - \bar{\chi}_m^T = \frac{2j\kappa}{\mu_0} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

For the twin-slab phase shifter, the tensor is

$$\bar{\chi}_m - \bar{\chi}_m^T = \pm \frac{2j\kappa}{\mu_0} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

where the sign change indicates the difference in the polarity of $H_{dc}$ between the two ferrite slabs [Fig. 1(c)]. The effect of the tensor operation is to transform the dot products in the integrand of (3) into a scalar triple product. In the case of the Faraday rotators, the integrand becomes proportional to

$$H^+ \times H^- \cdot a_z \, dS$$

and for the twin-slab phase shifter the integrand is proportional to

$$H^+ \times H^- \cdot a_y \, dS$$

where $a_y$ and $a_z$ are directional unit vectors.

Solutions of (2) for the magnetic loss factor $L$, (3) for the normalized phase shift $\phi/\phi_0$, and (6) for the loss ratio $r$ are summarized in Table I for the three configurations in Fig. 1. Because of the similarity between trigonometric and Bessel functions, the numerical results for the square and circular Faraday rotators are not significantly different (deviations do not exceed 5 percent for calculated values and are generally less than the resolution possible on the graphs presented here). Therefore, the results for the circular Faraday rotator are not shown graphically.

Figs. 2 and 3 show the figure of merit versus filling ratio for the square Faraday rotator and the twin-slab phase shifter, with the loss ratio $r$ as a parameter. Note that when $r = 0$, the figure of merit $F$ is just equal to the normalized phase shift $\phi/\phi_0$. (For the twin-slab phase shifter the figure of merit is not a function of the normalized operating frequency $f/f_c$. This is a consequence of the particular geometry under consideration and would not be

TABLE I
THEORETICAL SOLUTIONS

Square Faraday rotator:

$x = (\pi/2)b/a$ $\quad$ $f$ = operating frequency

$T = \tan(x)$ $\quad$ $S = \dfrac{\sin(2x)}{2x}$ $\quad$ $f_c$ = cutoff frequency

$$L = 0.405x^2\{1 + [1 - 2(f_c/f)^2]S\}$$

$$\phi/\phi_0 = \sin^2(x)$$

$$r = \frac{2.47}{x}\{T - x + [0.5 - (f_c/f)^2]T(S-1)\}^{-1}$$

Circular Faraday rotator: ($f, f_c$ as defined above)

$J_n$ = $n$th-order Bessel function

$x = s_{11}b/a$, where $s_{11}$ is the first root of $J_1$

$$B = J_1(x)/J_0(x)$$

$$L = \frac{J_0^2(x)}{0.809}[x(1+B^2) - 2B^2 + 2(f_c/f)^2(B^2 - Bx)]$$

$$\phi/\phi_0 = \frac{J_1^2(x)}{0.339} \quad \text{(this result was also obtained by Severin [5])}$$

$$r = \frac{0.809J_1'(x)}{J_0^3(x)} \cdot \{-x^2(1+B^2) + x(2B + B^3) + (f_c/f)^2[x(B+B^3) - 2B^2]\}^{-1}$$

Twin-slab phase shifter:

$$x = \pi t/a$$

$$L = 0.637x$$

$$\phi/\phi_0 = \sin(x)$$

$$r = \frac{1.571}{\tan(x) - x}$$

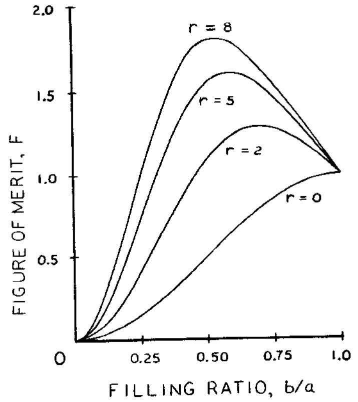


Fig. 2. Figure of merit versus filling ratio, square Faraday rotator ($f/f_c$ = 1.2).

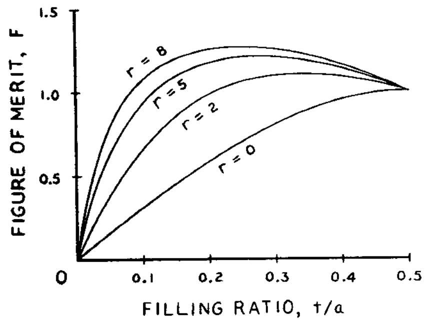


Fig. 3. Figure of merit versus filling ratio, twin-slab phase shifter.

true if, for example, the slabs were positioned against the side walls of the waveguide.) The improvement in the figure of merit for the Faraday rotator is greater than for the twin-slab phase shifter. This is a result of the fact that the magnetic loss factor $L$ varies with approximately the square of the filling ratio in the former case and linearly in the latter (i.e., in all cases $L$ is closely proportional to the cross-sectional area of the ferrite). The optimum filling ratios and maximum figures of merit as functions of the loss ratio are illustrated in Figs. 4–7. As discussed in Section III-A, the improvement in the performance with partial filling increases as cutoff is approached. This is evident in Fig. 5 for the Faraday rotator.

## V. EXPERIMENTAL RESULTS

Experiments were conducted at $X$ band using a fully filled and a partially filled ($b/a$ = 0.635) Faraday rotator of circular cross section. The ferrite was a commercially available magnesium manganese type with a relative dielectric constant of 12.7. The dielectric sleeve was an alumina tube with a relative dielectric constant of 9.2. (Tubes with dielectric constants more closely matched to the ferrite were not available to the authors.) Waveguides were formed by application of a fired glass and silver conductive composition.

In order to determine phase shift and loss characteristics, the test pieces were treated as transmission-line cavity resonators with extremely loose coupling. Loss information was obtained from the shape of the cavity resonance and phase shift was obtained from the shift in

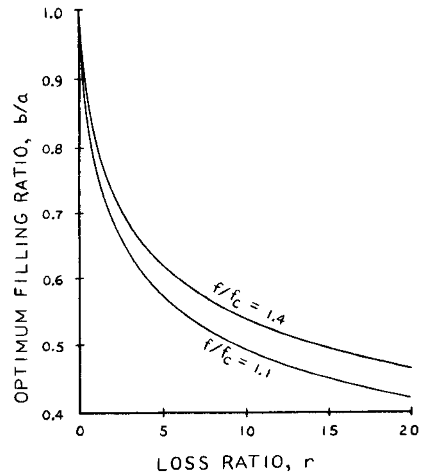


Fig. 4. Optimum filling ratio versus loss ratio, square Faraday rotator.

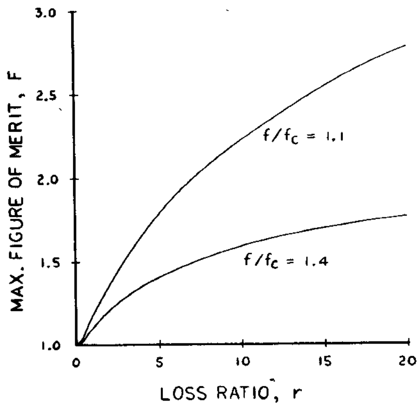


Fig. 5. Maximum figure of merit versus loss ratio, square Faraday rotator.

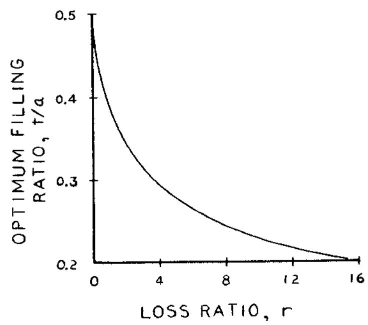


Fig. 6. Optimum filling ratio versus loss ratio, twin-slab phase shifter.

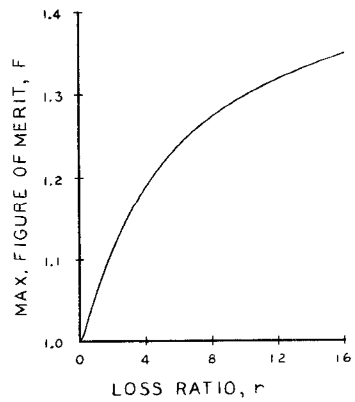


Fig. 7. Maximum figure of merit versus loss ratio, twin-slab phase shifter.

TABLE II
EXPERIMENTAL RESULTS—FARADAY ROTATOR

| | |
|---|---|
| Fully filled: $a = 0.82$ cm | |
| | $\alpha_0 = 0.094$ dB/cm |
| | $\phi_0 = 14°$/cm |
| | $\phi_0/\alpha_0 = 150°$/dB |
| Partially filled: $a = 0.80$ cm | $b/a = 0.635$ |
| | $\alpha = 0.048$ dB/cm |
| | $\phi = 16.5°$/cm |
| | $\phi/\alpha = 345°$/dB |
| Figure of merit: $F = \dfrac{\phi/\alpha}{\phi_0/\alpha_0} = 2.3$ | |

resonant frequencies with applied $H_{dc}$.[1] The results of the experiments are summarized in Table II. Note that the phase shift for the partially filled unit increased with respect to the fully filled unit, while theory predicts a decrease of about 30 percent (Fig. 2, $r = 0$ curve). Based on the predicted decrease in phase shift, the figure of merit should be approximately 1.4, while a value of 2.3 was found experimentally. The increase in phase shift is most likely a result of enhancement of the Faraday rotation because of the difference in the dielectric constants of the sleeve and ferrite rod. That result, together with the decrease in loss by almost 50 percent, may make the alumina and ferrite combination very appealing for low-loss and lightweight phase shifters.

## VI. CONCLUSIONS

It has been shown theoretically that when magnetic losses in the ferrite constitute an appreciable fraction of the total loss for ferrite phase shifters, a significant improvement in the figure of merit may result from partial filling of the structure. The limitation to this simple theory is that a uniform dielectric constant is assumed throughout the waveguide, which may be difficult to achieve in practice. The advantages of a partially filled Faraday rotator consisting of a ferrite rod inside an alumina tube have been demonstrated experimentally.

The partial filling technique that has been presented would probably be most useful for high-power applications where ferrite material with a high threshold must be used, at the expense of added magnetic loss. Under these conditions it may be required to minimize the overall loss in the phase shifter for thermal considerations. For other applications this technique would be of limited usefulness, since low-loss ferrites are readily available and the net reduction in insertion loss may not justify the added complexity and cost of partial filling.

[1] No attempt was made to magnetically saturate the material to achieve maximum phase shift. Rather, low and constant (between samples) applied bias fields were used to determine relative phase shifts. In practice, phase shifts in excess of 120°/cm may be obtained at $X$ band.

## ACKNOWLEDGMENT

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